HW5. Computing with TMs

2.18.25

Rules

- **Annotation:** all machines must be annotated.
- **Arguments:** Tally problems are defined on arguments $x \geq 1$; for binary problems, $x \geq 0$.
- **Blocks:** blocks are define differently for tally and binary.
  - Tally block = a single block of 1's.
  - Binary block = a block of 0's or 1's, flanked by *s.
- **X- and Y-Blocks:**
  - For 1-place functions, the tape starts with a single block.
    For 2-place functions, the tape starts with two blocks.
  - When a problem involves two arguments x and y:
    - The x-block is on the **right**, and the y-block is on the **left.**
- **Standard Position.**
  - All problems begin in standard position, and must return to standard position.
    - Tally SP = the right most cell of the right most block.
    - Binary SP = the right most * of the right most block.
- **Final configuration:**
  - All problems must end in standard position.
  - All problems must end with a well-formed output: a single block.
- **Boxes**
  - You can use a boxed version of any problem you have solved in an earlier problem, provided it satisfies the criteria for boxing.

I. TM Arithmetic: Tally

Assume $x,y \geq 1$.

1. $x+1$ T

2. $x+3$ T.
   Use your solution from (1).

3. $3x$ T.
   Use your solution from (2).

4. $x+y$ T.
   Use your solution from (1).
   For this problem, do not assume that the input blocks will be only separated by 1 cell. They may be separated by any number of cells.

5. $3(x + y)$ T.
   Use your solutions from (3) and (4).

6. $x + 3y$ T.
   Use your solutions from (3) and (4).

II. TM Arithmetic: Binary

Assume $x,y \geq 0$. The following problems have some arbitrary constraints built-in to make them simpler.

7. $x+1$ B
   • Assume the input block will always have an extra 0 in the left-most cell.

8. $x - 1$ B
   $f(x) = x - 1$ if $x \geq 1$
   $f(x) = 0$ if $x = 0$

9. $x + y$ B
   Use your solutions from (7) and (8).
   • Assume that y-block contain enough 0's padding the left side initially so that the entire block may contain the binary representation of $x+y$.
   • Don't forget to "clean up" your tape at the end of computation.

III. Algorithmic Design

10. Multiplication: $x \cdot y$

    Using a combination of flow charts, prose, and "snapshots" of the tape, describe a step-by-step *strategy* for computing multiplication. This should not be a complete Turing Machine. (That is a difficult and laborious problem.) But it should be sufficiently detailed that you think someone *could* build a Turing Machine from your instructions, if they knew what they were doing. Your choice of Tally or Binary.

11. Multi-function machine

    Design a machine which can perform different functions depending on the user input (at least successor, addition, multiplication), similar to a calculator. There are many way to do this, so be creative. Use a combination of flow charts, prose, and "snapshots" of the tape; don't design a completed Turing Machine.